The Mendeleev–Seaborg Periodic Table: Through Z = 1138 and Beyond

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There are many choices for displaying periodicity in the chemical elements (*1*). A straightforward display is arguably the full long-form arrangement shown in Figure 1, which includes very recent claims as high as element 118, eka-radon. The common version of the long form offsets the lanthanides and actinides to a footnote-like appearance, facilitating presentation on a full page. The title of this paper implies a discussion of extending the periodic table to Z = 1138 (and beyond). Being on the verge of starting a new row in the table, one needs to address such problems as how much further it will go and what to do about the format of the display. The intent is for this to be an interesting, informative, entertaining, and yet relatively simple exploration of those questions.

Choosing the arrangement in Figure 1 allows one to spot the valence electron structure from which chemical and physical properties can be inferred. The pattern shows areas characteristically corresponding to the s- and p-block main group (or representative) elements, the d-block transition elements, and the f-block inner transition elements. The last contains the lanthanides and the actinides.[1] By completing the 7th row with element 118, subsequent elements start the 8th row with eka-francium. Following the same chemical intuition that Glenn Seaborg applied in 1944 to Mendeleev's periodic table when he realized that the actinide elements comprise the second row of f-block elements, one should not be surprised with arguments that a g-block could commence the "super-actinides" with element ~121 (*2, 3*).

Are there opposing points of view? Yes, one is obliged to face at least two issues: relativity and nuclear stability. The former, in simple terms, is recognition that electrons close to the highly charged nucleus are moving at speeds approaching that of light. Important relativistic effects need to be accommodated. The second phenomenon involves the legitimate question of whether nuclei with $Z > \sim 121$ live long enough to acquire atomic electrons and hence be classifiable as atoms with reserved places on the periodic table.

Relativistic Effects

One can quickly get a rough estimate of the average speed of a 1s electron using Bohr's model of a one-electron system. The radius of Bohr's innermost planetary orbits—for the K shell—varies as a_0/Z, where Z is the atomic number (nuclear charges) and a_0 is the Bohr radius constant (53 pm). The radius can be used, for example, in conjunction with Bohr's hypothesis of quantized angular momentum, $m_e vr = nh/2\pi$, in which m_e is the electron mass (9.11×10^{-31} kg) and h is Planck's constant (6.63×10^{-34} J s) to give $|v| \approx 2.2 \times 10^6 Z$ m/s. One notes that the K-electron "speed" is about 10% the speed of light already for elements before the 3rd row has been completed. Despite the impropriety of Bohr's model, the estimation unmistakably indicates that relativistic effects become quite important for heavier elements. Indeed, it has been pointed out in this *Journal* (*4*) that the color of gold, the liquid nature of mercury, and anomalies in electron configurations are due to such considerations. Already for rutherfordium (Z = 104) and dubnium (Z=105), chemical behavior differs from that of their lighter homologues (*5*). An easily followed, more thorough discussion of relativistic effects can be found in this *Journal* (*6*). In fact, as one thinks about elements encroaching into the g-block, perhaps it would be no surprise that the configurations begin to shuffle seemingly erratically, muddling the periodic nature of chemical behavior. What to do then?

It would be most reasonable and practical to maintain recourse to the existing logic behind the Mendeleev–Seaborg periodic table. That is, regardless of what valence electron configurations emerge, the sequence of elements by atomic number should be exhibited in what could be called a shell-partitioned display format (the "spdf" format), which would extend it to the g-, h-, i-block elements and so on. The spdf Mendeleev–Seaborg periodic table is illustrated in Figure 2, carrying it through the element Z = 1138 in the lower right corner. Since Kr, Xe, and Rn become more and more

s-block		f-block														d-block										p-block					
H																															He
Li	Be																									B	C	N	O	F	Ne
Na	Mg																									Al	Si	P	S	Cl	Ar
K	Ca															Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Ae															Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	Rf	Db	Sg	Bh	Hs	Mt	110	111	112		114		116		118

Figure 1. The full long-form periodic table of the elements.

chemically reactive and have higher and higher melting points, the hypothetical element 1138 would plainly be expected to be a not-so-noble solid, and very dense.

Nuclear Stability and Instability

Addressing the size of the periodic table indirectly questions the stability of the atomic nucleus as one goes to higher and higher elements. There is a very elegant model of nuclear stability that has been successful in describing the nucleus as comparable to a liquid drop containing A (mass number) constituents: neutrons plus protons, collectively referred to as *nucleons*. A brief discussion of this model follows.

Because of the short-range nature of the nuclear force, qualitatively analogous to that of intermolecular attractions, the average binding energy per constituent particle, E_B/A, would be constant for an infinitely large drop. The density would be constant and the volume would be $\propto A$ (implying the radius is $\propto A^{1/3}$). For a drop with a finite number of units (nucleons), there would be a correction for particles on the surface corresponding to the surface tension familiar in liquids. The number of such particles is proportional to the surface area of the drop, that is, $\propto A^{2/3}$. All other things being equal, surface tension would minimize the drop's energy at a geometry corresponding to a spherical shape. For a nuclear drop, one also has a repulsive contribution to the total energy arising in the mutual electrostatic interaction among the Z protons. This contribution varies with $Z(Z - 1)$ or, for large numbers, approximately as Z^2 to account for the number of such interactions, and varies inversely as the distance between protons in the nucleus. The latter is proportional to the nuclear radius, $r_0A^{1/3}$, where r_0 is a radial scaling constant (1.2×10^{-15} m). The picture is now that of a *charged* liquid drop. The most stable situation would be a spherical nucleus containing no protons (i.e., all neutrons).

Obviously, something has not yet been accommodated in the model. That omission is a quantum mechanical effect that gives rise to a *symmetry energy* correction. Both neutrons and protons, like electrons, obey the Pauli exclusion principle: two neutrons per neutron state, two protons per proton state when filled. If corresponding neutron and proton quantum states are approximately equal in energy, equal numbers of neutrons and protons would always be more stable than a configuration that had more of one than the other. This is because the excess type could lower its energy by beta-decaying into the type in deficit. That is, a great excess of neutrons over protons could become more stable by emitting (negative) betas, becoming protons, and vice versa, moving the configuration back toward a symmetric composition and releasing energy. The symmetry effect argues that all stable nuclei should have $N \approx Z$ (or $Z \approx A/2$). But this isn't the case either. Putting all the pieces together gives the expression for the total nuclear binding energy E_B below, the terms of which correspond to the order of discussion above.

$$E_B = c_{vol}A - c_{surf}A^{2/3} - c_{coul}Z^2A^{-1/3} - c_{sym}(A - 2Z)^2A^{-1}$$

Values for the constants c are in principle derivable from the theory of nuclear forces. Such theories turn out to be much, much more complicated than that of the electrostatic force needed to discuss electron behavior in atoms and molecules (although enormous progress has been made in this area over the past several decades). However, a simple alternative is to take the historical approach of using the binding-energy equation in a semiempirical way, applying it to measured values of E_B and extracting the best-fit constants. The approximate values, yielding errors in E_B of <1%, are

$$c_{vol} \approx 14 \text{ MeV}$$

$$c_{surf} \approx 13 \text{ MeV}$$

$$c_{coul} \approx 0.6 \text{ MeV}$$

$$c_{sym} \approx 20 \text{ MeV}$$

For a fixed mass number A, the most stable nucleus can be approximated by maximizing[2] E_B with respect to Z at constant A. The result is that for light nuclei, where not too much electrostatic repulsion has accumulated, the *sym*metry term dominates and $N \approx Z$ (e.g. ^{16}O, ^{24}Mg). As Z begins to increase, the *coul*omb term becomes more and more influential so that $N > Z$. At the lanthanide elements, $N/Z \approx 1.5$; that is, there is a 50% neutron excess over protons for nuclei stable toward beta decay (e.g. ^{169}Tm) and the excess is even larger for the newer, transactinide elements. Typical E_B/A for known stable elements is ≈8 MeV per nucleon. From what has been presented, the trend should continue and, for example, the most beta-stable isotope of element 1138 would have a mass number of about 8720 corresponding to an N/Z ratio of 6.7. Such behemoths have not been considered. The reasons for this will be demonstrated once further discussion of stability has been provided.

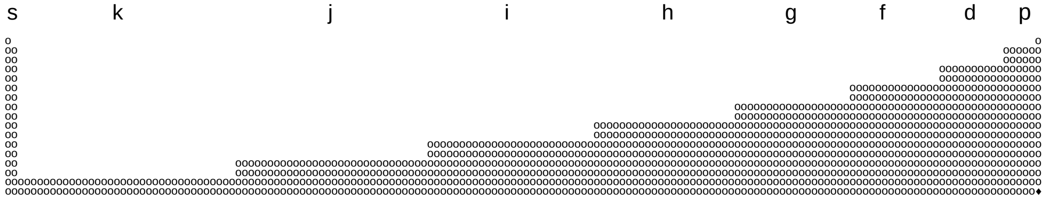

Figure 2. The extended "spdf" periodic table after Mendeleev and Seaborg showing the various blocks through element Z = 1138, the latter indicated by the black location in the lower right corner.

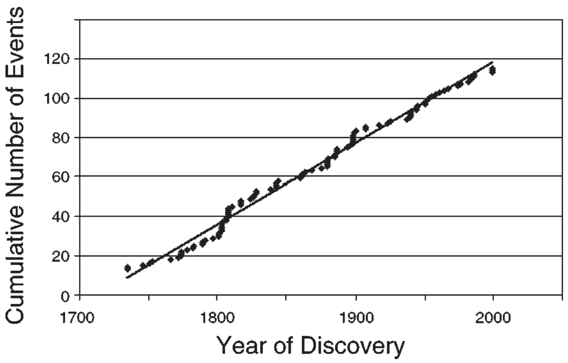

Figure 3. The cumulative number of known elements (not sequential) as a function of year of discovery during the past two-and-a-half centuries. The line is a linear regression fit with a slope corresponding to one new element every 2.5 years.

A very quick look at describing the composition of the most stable nuclei has been given. But that is not to imply that these are stable in the absolute sense. Uranium has already been used to bombard uranium, a fusion combination that hypothetically leads to element $Z = 184$. Yet this superheavy element hasn't been found. The rate at which new elements have been discovered over the past quarter millennium during which the science of chemistry has matured is illustrated in Figure 3. The rate has been almost perfectly constant (as indicated by the straight line[3]) at one new element (not sequential) every 2.5 years. A tongue-in-cheek estimate is that it will be more than another century before completion of the 8th row on our table is achieved.

However, the issue with heavier elements is not one of improving technology, but rather the rapidly diminishing time of survival of the nucleus. Why? Because here fission occurs virtually instantaneously. At higher and higher atomic numbers, the possibility of fission becomes unavoidable.[4] Within the simple liquid-drop model, the phenomenon of fission is viewed as the result of the drop's distorting from its spherical shape. To do this, it must overcome surface tension. The push to distort originates in the diminution of the electrostatic repulsion as large clusters containing protons (and neutrons) pull apart.[5] In this *Journal*, Huizenga (*7*) illustrated that the balance between these opposing shapes tips over to favor fission when $Z^2/A = 2c_{\mathrm{surf}}/c_{\mathrm{coul}}$. The average values indicated above give $2c_{\mathrm{surf}}/c_{\mathrm{coul}} \approx 43$. From experimental data for just the heaviest known, most stable elements, $2c_{\mathrm{surf}}/c_{\mathrm{coul}} \approx 50$ and $A/Z \approx 2.5$. From these magnitudes, one infers that the liquid-drop model for elements beyond $Z \approx 125$ corresponds to a situation where $Z^2/A > 50$; that is, the isotopes most stable toward beta decay nevertheless *are unstable toward instantaneous breakup by fission*. For $Z = 184$, the "most beta-stable" mass number is ≈550 (currently unreachable); $A/Z \approx 3$; $Z^2/A \approx 60$; and $E_B/A \approx 5.7$ MeV per particle.[6] For our most stable $Z = 1138$ isotope, $Z^2/A \approx 150$, way beyond the instantaneous fission threshold of $Z^2/A \approx 50$.

This is discouraging. Not surprisingly, few have explored what happens if one keeps on going…hypothetically. Yet, according to this presentation's title, a discussion of "element 1138 and beyond" is forthcoming. How can one talk about even being much beyond element 125? Guess what! As nuclei get still heavier and heavier, the proportion of neutrons keeps on increasing. By $A \approx 1{,}700{,}000$, the most stable element has $Z \approx 9000$ giving—once again—$Z^2/A \approx 50$, and implying a return to elements not only stable against beta decay but also capable of resisting the fission breakup path. However, there is another problem: the binding energy per particle, in the absence of any shell considerations, has dropped significantly, leading to the possibility that neutrons are no longer bound to the conglomerate. If the binding energy of neutrons to a nucleus goes to zero, one has reached what is called the *neutron drip line*. This is the composition at which neutrons, as the label implies, drip spontaneously off the nucleus. In fact, if you take the simple binding energy equation above at face value for the $Z \approx 9000$ system, E_B/A would be ≈−1.9 MeV per nucleon, implying that the drip line has been breached. The macronucleus would literally hemorrhage neutrons. Of course, it is far and away unreasonable to expect such systems to be reached by assemblage from lighter conglomerates. But what about from heavier conglomerates?

Supermassive Ensembles

The question is not facetious. Yet another term must be included in the liquid-drop expression. The term involves the contribution from a phenomenon we're actually most familiar with. Gravity! Gravity provides our final diversion.[7]

Under ordinary circumstances, E_{grav}, the mutual attraction among the nuclear components due to gravity, is so weak as to be completely, utterly neglectable. But eventually, as A increases to astronomically large values, its contribution becomes significant. The additional term would be $+c_{\mathrm{grav}}A^{5/3}$ and it originates in the following way. There are $A(A-1)/2$ gravitational attractions among the A nucleons, each with a particle mass $m_A = 1.7 \times 10^{-27}$ kg distributed uniformly in the volume of a sphere of radius $r_0A^{1/3}$. The gravitational energy (needed to separate the system into A individual pieces at infinite mutual separation) would be

$$\frac{3}{5r_0A^{1/3}}\,\frac{A(A-1)}{2}\,m_A^{\,2}G \approx \frac{0.3m_A^{\,2}G}{r_0}\,\frac{A^2}{A^{1/3}}$$

G is the universal gravitational constant, 6.7×10^{-11} Nm^2/kg^2. The coefficient c_{grav} is then ≈4.6×10^{-50} J (2.9×10^{-37} MeV). The charged liquid drop's electrostatic repulsion term c_{coul} is 2×10^{36} times larger and so will continue to dominate until the number of nucleons becomes colossal. The pivot point where fission is spontaneous now becomes $Z^2/A = 2c_{\mathrm{surf}}/c_{\mathrm{coul}} + Ac_{\mathrm{grav}}/c_{\mathrm{coul}}$. Investigating this extreme situation just a bit further will prove interesting. The first term in Z^2/A we've already noted is around 50. If A is of the order of 10^{38} the second term is then also around 50. That many nucleons would have a mass of 2×10^{11} kg. For reference, our sun, which is not nuclear but is a plasma, has a mass of 2×10^{30} kg. The mass of Jupiter is 2×10^{26} kg. The gravitational contribution to the fissionability constant Z^2/A eventually renders fission inconsequential in astronomical *nuclear* objects, whatever they are.

A neutron star typically has a mass comparable to that of our sun and, as the name implies, is mostly neutrons. Treating a *nuclear* cluster of that size with the binding energy expression, E_B, including gravitational energy, gives a stable composition of $N/Z \approx 10^{36}$ for element[8] $Z \approx 10^{21}$ and a binding energy per particle back up to $E_B/A \approx +24$ MeV per nucleon. That's very stable.

Neutron stars exist. They have traces of protons and electrons and are stable toward beta decay and to breakup by fission. Neutron stars have been around for a long time, arguably representing our heaviest elements…already in nature.

Summary

The periodic table is probably going to intrude upon the 8th row in the not-too-distant future. This brings up questions about how to extend the format of the table and how much further it might go. Regarding the first point, despite the probable scrambling of electron configurations via relativistic effects, the suggestion has been made to embrace the shell-partitioned display format—that is, the "spdf" style of the long-form Mendeleev–Seaborg periodic table. Regarding how far the table might be extended, the question of nuclear stability for larger and larger systems and the role of fission, which predict an end to stability and ordinary chemical expectations, was discussed from a purposely simplified perspective. Eventually the role of gravity grows in significance, leading to the viability of astronomically large stable nuclei, already present in nature as neutron stars.

Notes

1. The IUPAC Nomenclature of Inorganic Chemistry ("The Red Book", 1990) states that the lanthanides and actinides, containing La–Lu and Ac–Lr respectively, comprise the f-block elements. Whether one or the other end of both 15-element sequences should be really included remains controversial.

2. More accurately, the mass should be minimized.

3. The effects of new techniques, instrumentation, and production designs can be seen as systematic departures from the statistical fit. The time bunchings and gaps are discussed this *Journal* (*6*).

4. By keeping with just the very simple liquid-drop model, one admittedly neglects for heuristic purposes the all-important nuclear shell structure that imparts significant extra binding, often sufficient to inhibit breakup by fission at islands of stability (*2, 3*) corresponding to closed shells.

5. The volume and N/Z composition are kept constant during this fission picture.

6. The legitimacy of considering the chemistry of hypothetical elements with basically zero lifetimes is questionable, of course. Nevertheless, there is already a publication (Jiang, C-X. *Phys. Lett. A* **1979,** *73,* 385) that discusses the electronic structure of elements 172 and 226.

7. Again, in order not to distract from the simple qualitative demonstration, additional phenomena are purposely omitted. In this case, they involve exotic particles and increasing density.

8. The prefix recently adopted in chemistry and physics to represent 10^{21} is *zetta*. Perhaps our "element" should be called *zettium*. I prefer *godzillium*.

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